Decentralized Control of Petri Nets

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The goal of the paper is to extend the supervision based on place invariants (SBPI) to a decentralized setting.

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Overview of the Supervision Based on Place Invariants

*Supervision Based on Place Invariants:* introduced by several researchers (Giua, Yamalidou, Moody, and others).

The specification of the SBPI is $L\mu \leq b$.

**Case I: All transitions are controllable and observable.**

Let $D$ be the incidence matrix of the plant Petri net. The supervisor can be designed as a Petri net of incidence matrix

$$D_s = -LD$$

If $\mu_0$ is the initial marking of the plant, the initial marking of the supervisor is

$$\mu_{s0} = b - L\mu_0$$

The places of the supervisor are called *control places*. The closed-loop is a Petri net of incidence matrix

$$D_c = \begin{bmatrix} D \\ -LD \end{bmatrix}$$
Overview of the Supervision Based on Place Invariants

Example

The set of constraints

\[
\mu(p_1) + \mu(p_3) \geq 1 \\
\mu(p_2) + \mu(p_3) \geq 1
\]

is described by \( L\mu \leq b \) with:

\[
L = \begin{bmatrix}
-1 & 0 & -1 \\
0 & -1 & -1
\end{bmatrix} \quad b = \begin{bmatrix}
-1 \\
-1
\end{bmatrix}
\]

The incidence matrix is:

\[
D = \begin{bmatrix}
-1 & 1 & 0 \\
-1 & 0 & 1 \\
2 & -1 & -1
\end{bmatrix}
\]

The supervisor has two control places (as \( L \) has two rows):

\[
D_s = -LD = \begin{bmatrix}
1 & 0 & -1 \\
1 & -1 & 0
\end{bmatrix}
\]

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Overview of the Supervision Based on Place Invariants

The initial marking of the supervisor is

$$\mu_{s0} = b - L\mu_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Note that for all reachable markings

$$\mu_s = b - L\mu$$

This approach is called *supervision based on place invariants*, as it creates for each row of $L$ a place invariant. In particular:

- $$\mu(p_1) + \mu(p_3) - \mu(C_1) = 1$$
- $$\mu(p_2) + \mu(p_3) - \mu(C_2) = 1$$
Overview of the Supervision Based on Place Invariants

Case II: Not all transitions are controllable and observable.

A supervisor should not inhibit uncontrollable transitions or observe firings of unobservable transitions.

Then, the supervisory approach of Case I can still be used if (but not only if)

\[
LD_{uo} = 0 \quad \text{and} \quad LD_{uc} \leq 0
\]

(1)

where \(D_{uc}\) and \(D_{uo}\) are the restrictions of the incidence matrix \(D\) to the columns of the uncontrollable and unobservable transitions, respectively.

To enforce \(L \mu \leq b\) we can proceed as follows:

1. If \(L\) satisfies (1), find the supervisor as in Case I. Otherwise:
2. Transform \(L \mu \leq b\) to \(L_a \mu \leq b_a\) such that \(L_a \mu \leq b_a \Rightarrow L \mu \leq b\) and \(L_a\) satisfies (1). Then the supervised PN is obtained as in Case I by enforcing \(L_a \mu \leq b_a\) instead of \(L \mu \leq b\).
Overview of the Supervision Based on Place Invariants Example

Assume \( t_1 \) unobservable and the same specification:

\[
\begin{align*}
\mu(p_1) + \mu(p_3) &\geq 1 \\
\mu(p_2) + \mu(p_3) &\geq 1
\end{align*}
\]

\[
L = \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \end{bmatrix} \quad b = \begin{bmatrix} -1 \\ -1 \end{bmatrix}
\]

As \( D = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & -1 & -1 \end{bmatrix} \), \( D_{uo} = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} \) and \( D_{uc} \) is empty.

Note that \( LD_{uo} = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \neq 0 \).

Therefore, the constraints are transformed to

\[
\begin{align*}
2\mu(p_1) + \mu(p_3) &\geq 1 \\
2\mu(p_2) + \mu(p_3) &\geq 1
\end{align*}
\]

and enforced by the control places \( C_1 \) and \( C_2 \).

\( t_1 \) is unobservable

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3. Decentralized Admissibility

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Centralized vs Decentralized

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Decentralized Supervision

Example 1

\[ T_{c,1} = T_{o,1} = \{t_1, t_2\} \]

\[ T_{c,2} = T_{o,2} = \{t_3, t_4\} \]

Specification: \( \mu_1 + \mu_3 \leq 1 \)
Decentralized Supervision

Example 2

\[ T_{c,1} = T_{o,1} = \{t_1, t_2\} \]

\[ T_{c,2} = T_{o,2} = \{t_3, t_4\} \]

\[ T_{c,3} = \{t_5\} \]

\[ T_{o,3} = \{t_1, t_2, t_3, t_4, t_5\} \]

Specification:

\[ \mu_1 + \mu_3 \leq 1 \]

\[ \mu_5 \leq 1 \]

\[ \mu_6 \leq 1 \]
Decentralized Supervision

Given:

- the Petri net model of the system
- the sets of controllable and observable $T_{c,i}$ and $T_{o,i}$, $i = 1 \ldots p$.
- the specification $L \mu \leq b$.

**Problem 1:** Find the supervisors $S_1 \ldots S_p$ such that

1. The joint operation of $S_1 \ldots S_p$ ensures the plant satisfies $L \mu \leq b$.
2. Each $S_i$ controls only transitions in $T_{c,i}$ and observes only transitions in $T_{o,i}$.
**Decentralized Supervision with Communication**

**Problem 2:** Solve Problem 1 when communication is allowed.

Communication can be used to enable $S_i$ to

- control $t \in \bigcup_{j \neq i} T_{c,j}$, $t \notin T_{c,i}$.
- observe $t \in \bigcup_{j \neq i} T_{o,j}$, $t \notin T_{o,i}$.

**Remark:** Centralized supervision assumes:

$$T_{c} = \bigcup_{j=1\ldots p} T_{c,j} \text{ and } T_{o} = \bigcup_{j=1\ldots p} T_{o,j}$$

that is, full (maximum) communication!

Optimality criteria:

- minimum communication.
- maximally permissive design.
The goal of the paper is to extend the supervision based on place invariants (SBPI) to a decentralized setting.

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Decentralized Admissibility

In centralized supervision:

- it is (computationally) easy to enforce constraints $L\mu \leq b$ on fully controllable and observable PNs.
- in partially controllable and observable PNs, we say that $L\mu \leq b$ is \textit{c-admissible} if it can be enforced as if the PN were fully controllable and observable.
- constraints $L\mu \leq b$ that are not c-admissible are transformed to a c-admissible form $L_a\mu \leq b_a$ such that $L_a\mu \leq b_a \implies L\mu \leq b$.

In decentralized supervision:

- we extend c-admissibility to \textit{d-admissibility}, such that
  - d-admissible constraints $L\mu \leq b$ are (computationally) easy to enforce.
  - checking whether a set of constraints is d-admissible is (computationally) tractable.
- the definition we propose allows us to
  - transform constraints $L\mu \leq b$ that are not d-admissible to d-admissible constraints $L_a\mu \leq b_a$ such that $L_a\mu \leq b_a \implies L\mu \leq b$.
  - enforce constraints that are not d-admissible by enabling communication
Decentralized Admissibility

Let $L \mu \leq b$, $L \in \mathbb{Z}^{m \times |P|}$ and $b \in \mathbb{Z}^{m \times 1}$ be a set of constraints. A constraint of $L \mu \leq b$ is denoted by $l \mu \leq c$, $l \in \mathbb{Z}^{1 \times |P|}$ and $c \in \mathbb{Z}$.

$l \mu \leq c$ is **d-admissible** with respect to $(\mathcal{N}, \mu_0, T_c, 1 \ldots T_c, n, T_o, 1 \ldots T_o, n)$, if there is $C \subseteq \{1, 2, \ldots, n\}$, $C \neq \emptyset$, such that $l \mu \leq c$ is c-admissible with respect to $(\mathcal{N}, \mu_0, T_c, T_o)$, where $T_c = \bigcup_{i \in C} T_{c,i}$ and $T_o = \bigcap_{i \in C} T_{o,i}$.

$L \mu \leq b$ is **d-admissible** if each of its constraints $l \mu \leq c$ is d-admissible.

- c-admissibility is a special case of d-admissibility, in the sense that if $l \mu \leq c$ is c-admissible w.r.t. $(\mathcal{N}, T_{c,i}, T_{o,i})$, $l \mu \leq c$ is d-admissible (set $C = \{i\}$).
- $l \mu \leq c$ d-admissible implies
  - If firing a plant-enabled transition $t$ violates $l \mu \leq c$ then $\exists i \in C$: $t \in T_{c,i}$.
  - All supervisors $S_i$ with $i \in C$ are able to know the value of $c - l \mu$.
- an algorithm checking whether a set of constraints is d-admissible is in the paper.
The goal of the paper is to *extend the supervision based on place invariants (SBPI) to a decentralized setting*

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Enforcement of D-admissible Constraints

Let $D$ and $\mu_0$ be the incidence matrix and the initial marking of a PN $\mathcal{N}$.

Recall the centralized enforcement of a c-admissible constraint $l\mu \leq c$ on $(\mathcal{N}, \mu_0)$:

- A control place $C$ is generated such that for all $t$:
  1. If $lD(\cdot, t) > 0$, then $C \in \bullet t$ and the weight is $W(C, t) = lD(\cdot, t)$.
  2. If $lD(\cdot, t) < 0$, then $C \in t\bullet$ and the weight is $W(t, C) = -lD(\cdot, t)$.
- The initial marking of $C$ is $c - l\mu_0$.

In the decentralized enforcement of a d-admissible constraint $l\mu \leq c$, for all $i \in C$:

- Define $x_i \in \mathbb{N}$, as the state variable of $S_i$.
- Initialize $x_i$ to $c - l\mu_0$.
- $S_i$ disables a transition $t$ if $t \in T_{c,i}$ and $x_i < lD(\cdot, t)$.
- If $t \in T_{o,i}$ fires and $lD(\cdot, t) \neq 0$, then $x_i = x_i - lD(\cdot, t)$.

It can be proved that the decentralized supervisor $\bigwedge_{i \in C} S_i$ enforces $l\mu \leq c$ and that it is equally permissive to the centralized supervisor $S$ enforcing $l\mu \leq c$ in the fully controllable and observable version of $\mathcal{N}$.
Enforcement of D-Admissible Constraints

Desired constraint: $\mu_1 + \mu_3 \leq 1$. Initial marking $\mu_0 = [0, 1, 1, 0]^T$.

Decentralized setting: $T_{c,1} = \{t_1, t_2\}$, $T_{c,2} = \{t_3, t_4\}$, $T_{o,1} = T_{o,2} = \{t_1, t_2, t_3, t_4\}$.

The supervisor $S_1$:
- initializes $x_1$ to 0.
- disables $t_1$ if $x_1 = 0$
- increments $x_1$ if $t_2$ or $t_3$ fires.
- decrements $x_1$ if $t_1$ or $t_4$ fires.

The supervisor $S_2$:
- initializes $x_2$ to 0.
- disables $t_4$ if $x_2 = 0$
- increments $x_2$ if $t_2$ or $t_3$ fires.
- decrements $x_2$ if $t_1$ or $t_4$ fires.

A graphical representation is possible, however it may be both helpful and misleading.
The goal of the paper is to
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Enforcement of D-Inadmissible Constraints via Communication

\[ \mu_1 + \mu_3 \leq 1 \] is d-inadmissible for \( T_{c,1} = T_{o,1} = \{t_1, t_2\} \) and \( T_{c,2} = T_{o,2} = \{t_3, t_4\} \).

The constraint becomes d-admissible if the transitions \( t_1 \) and \( t_2 \) are communicated to subsystem 2 and the transitions \( t_3 \) and \( t_4 \) to subsystem 1.

Then \( T_{o,1} = T_{o,2} = \{t_1, t_2, t_3, t_4\} \), \( T_{c,1} = \{t_1, t_2\} \) and \( T_{c,2} = \{t_3, t_4\} \).
D-inadmissible constraints can be made admissible by communication:

1. Let $T_{c,L} = \bigcup_{i=1}^{n} T_{c,i}$ and $T_{o,L} = \bigcup_{i=1}^{n} T_{o,i}$.

2. Is the specification c-admissible with respect to $(\mathcal{N}, T_{c,L}, T_{o,L})$? If not, transform it to be c-admissible.

3. Let $S$ be the centralized SBPI supervisor enforcing the specification. Let $T_c$ be the set of transitions controlled by $S$ and $T_o$ the set of transitions detected by $S$.

4. Find a set $C$ such that $\bigcup_{i \in C} T_{c,i} \supseteq T_c$.

5. The communication can be designed as follows: for all $t \in T_o \setminus (\bigcap_{i \in C} T_{o,i})$, a subsystem $j$ such that $t \in T_{o,j}$ transmits the firings of $t$ to all supervisors $S_k$ with $t \notin T_{o,k}$ and $k \in C$.

6. Design the decentralized supervisor according to the algorithm for d-admissible constraints.

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Enforcement of D-Inadmissible Constraints via Communication

In the algorithm

- No communication restrictions considered. These are considered later.
- The supervisor is equally permissive to the centralized supervisor.

In the communication policy proposed in the algorithm:

- The control decisions are taken locally (no control decisions are communicated).
- Assuming broadcast, there is less communication traffic than in the centralized solution, which remotely observes and controls the transitions in $T_o$ and $T_c$, respectively.
- Better communication policies may be possible. (The optimal policy can be obtained by solving an integer program.)
Enforcement of D-Inadmissible Constraints via Communication

$\mu_1 + \mu_3 \leq 1$ is d-inadmissible for $T_{c,1} = T_{o,1} = \{t_1, t_2\}$ and $T_{c,2} = T_{o,2} = \{t_3, t_4\}$.

$T_{c,L} = T_{o,L} = \{t_1, t_2, t_3, t_4\}$; $\mu_1 + \mu_3 \leq 1$ is c-admissible w.r.t. $(N, T_{c,L}, T_{o,L})$.

$T_c$ and $T_o$ found from the centralized SBPI:

$T_c = \{t_1, t_4\}$  \hspace{1cm}  $T_o = \{t_1, t_2, t_3, t_4\}$

$C = \{1, 2\}$
Enforcement of D-Inadmissible Constraints via Communication

Centralized

Broadcast: $t_1$, $t_2$, $t_3$, and $t_4$.
Remotely control: $t_1$ and $t_4$.

Decentralized

Broadcast: $t_1$ and $t_2$.
Remotely control: —
Broadcast: $t_3$ and $t_4$.
Remotely control: —
Enforcement of D-Inadmissible Constraints via Communication

Still another solution ...

In general, several equally permissive and decentralized solutions are possible.

The optimal solution depends on the relative cost of broadcast/remote control.
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Enforcement of D-Inadmissible Constraints via Transformations

**Specification:** \( L \mu \leq b \) (d-inadmissible)

**Goal:** Find \( L_1 \mu \leq b_1 \ldots L_m \mu \leq b_m \) that are d-admissible such that

\[
(L_1 \mu \leq b_1 \land L_2 \mu \leq b_2 \land \ldots L_m \mu \leq b_m) \Rightarrow L \mu \leq b
\]  

(2)

**Remarks:**

- Each \( L_i \mu \leq b_i \) has a different set \( C_i \).
- The sets \( C_i \) are given.
- Any solution can be found if all sets \( C_i \) are given. If so, \( m = 2^n - 1 \).
- However, we could discard the sets \( C_i \) with \( T_o^{(i)} = \bigcap_{i \in C_i} T_{o,i} = \emptyset \).
- In practice, we expect most sets \( C_i \) to have \( T_o^{(i)} = \emptyset \).

We propose to simplify (1) to:

\[
[(L_1 + L_2 + \ldots L_m) \mu \leq (b_1 + b_2 + \ldots b_m)] \Rightarrow L \mu \leq b
\]  

(3)
Enforcement of D-Inadmissible Constraints via Transformations

The following parametrization is used:

\[ L_1 + L_2 + \ldots + L_m = R_1 + R_2L \]  \hfill (4)

\[ b_1 + b_2 + \ldots + b_m = R_2(b + 1) - 1 \]  \hfill (5)

for \( R_1 \in \mathbb{N}^{m \times |P|} \), \( R_2 \in \mathbb{N}^{m \times m} \) such that \( R_2 > 0 \) and \( R_2 \) is diagonal.

Admissibility constraints

\[ L_i D(\cdot, T^{(i)}_{uc}) \leq 0 \]  \hfill (6)

\[ L_i D(\cdot, T^{(i)}_{uo}) = 0 \]  \hfill (7)

where \( T^{(i)}_{uc} = \bigcap_{i \in C_i} T_{uc,i} \) and \( T^{(i)}_{uo} = \bigcup_{i \in C_i} T_{uo,i} \).

Then the problem is to find a feasible solution of (4–7). The unknowns are \( R_1, R_2, L_i, \) and \( b_i \), and integer programming can be used to find them.

**Drawbacks:** The computational complexity of ILP and the fact that a permissivity requirement seems rather hard to be encoded as linear constraints.
Example

Specification: $\mu_1 + \mu_3 \leq 2$; $T_{c,1} = T_{o,1} = \{t_1, t_2\}$ and $T_{c,2} = T_{o,2} = \{t_3, t_4\}$.

Take $m = 2$, $C_1 = \{1\}$ and $C_2 = \{2\}$.

Decentralized solution: $\mu_1 \leq 1$ (as $L_1 \mu \leq b_1$) and $\mu_3 \leq 1$ (as $L_2 \mu \leq b_2$).
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Restricted Communication

The previous ILP approach can be used with communication extensions.

Note that:
- Communication allows improving permissivity.
- Some constraints are not enforceable without communication.

Extensions:

- The binary variables $\alpha_{ij}$ and $\varepsilon_{ij}$ are introduced.
  - $\alpha_{ij} = 1$ iff the firing of $t_j$ is announced to the supervisors of $C_i$.
  - $\varepsilon_{ij} = 1$ iff a supervisor from $C_i$ remotely controls $t_j$.
- In particular, in the broadcast case
  - $\alpha_{ij} = \alpha_j \forall i = 1 \ldots m$ ($\alpha_j = 1$ iff each firing of $t_j$ is broadcast, i.e., all supervisors are announced when $t_j$ fires).
  - $\varepsilon_{ij} = \varepsilon_j \forall i = 1 \ldots m$ ($\varepsilon_j = 1$ iff all supervisors are allowed to remotely control $t_j$).
- Communication constraints can be incorporated as expressions of $\alpha_{ij}$ and $\varepsilon_{ij}$.
Restricted Communication

- Define $B_U^i$ and $B_L^i$ as upper and lower bounds of $L_i D$.
- Let $A = [\alpha_{ij}]$ and $E = [\varepsilon_{ij}]$.

The admissibility constraints $L_i D(\cdot, T_{uo}^{(i)}) \leq 0$ and $L_i D(\cdot, T_{uo}^{(i)}) = 0$ are replaced by:

\[
L_i D(\cdot, T_{uo}^{(i)}) \leq [B_U^i \text{diag}(A(i, \cdot))]_{T_{uo}^{(i)}}
\]

(8)

\[
L_i D(\cdot, T_{uo}^{(i)}) \geq [B_L^i \text{diag}(A(i, \cdot))]_{T_{uo}^{(i)}}
\]

(9)

\[
L_i D(\cdot, T_{uc}^{(i)}) \leq [B_U^i \text{diag}(E(i, \cdot))]_{T_{uc}^{(i)}}
\]

(10)

Given the weight matrices $C$ and $F$, the objective of the ILP can be set to

\[
\min_{A, E, L_i, b_i, R_1, R_2} \text{Trace}(CA + FE)
\]

(11)

to minimize communication.

$C/F$ may reflect statistics on how often the transitions $t_j$ are fired/require control.
Manufacturing Example (Adapted from [Lin, 1990])

Machines: $M_1$ and $M_2$.
Buffers: $B_1 \ldots B_4$.
Robots: $H_1 \ldots H_4$.

Two possible manufacturing sequences:
- $\gamma_1 \tau_1 \pi_1 \alpha_3 \tau_3 \pi_3 \alpha_1 \eta_1$
- $\gamma_2 \tau_4 \pi_4 \alpha_2 \tau_2 \pi_2 \alpha_4 \eta_2$

$B_1$ and $B_2$ share common buffer space.
$B_3$ and $B_4$ share also common space.
Decentralized Supervision

\[ T_{c,1} = \{t_2\} \quad T_{o,1} = \{t_2, t_3, t_4\} \]
\[ T_{c,2} = \{t_5\} \quad T_{o,2} = \{t_5, t_6, t_7, t_8\} \]
\[ T_{c,3} = \{t_{10}\} \quad T_{o,3} = \{t_{10}, t_{11}, t_{12}\} \]
\[ T_{c,4} = \{t_{13}, t_{16}\} \quad T_{o,4} = \{t_{13}, t_{14}, t_{15}, t_{16}\} \]

Avoid buffer overflow: \( \mu_3 + \mu_{13} \leq 4 \) and \( \mu_6 + \mu_{10} \leq 4 \).

Take \( m = 4 \) and \( C_i = \{i\}, i = 1 \ldots 4 \).

Solution without communication:
\[ \mu_2 + \mu_3 \leq 2 \quad \text{(sub-1)} \]
\[ \mu_5 + \mu_6 \leq 2 \quad \text{(sub-2)} \]
\[ \mu_9 + \mu_{10} \leq 2 \quad \text{(sub-3)} \]
\[ \mu_{12} + \mu_{13} \leq 2 \quad \text{(sub-4)} \]
Decentralized Supervision

\[ T_{c,1} = \{t_2\} \]
\[ T_{c,2} = \{t_5\} \]
\[ T_{c,3} = \{t_{10}\} \]
\[ T_{c,4} = \{t_{13}, t_{16}\} \]
\[ T_{o,1} = \{t_2, t_3, t_4\} \]
\[ T_{o,2} = \{t_5, t_6, t_7, t_8\} \]
\[ T_{o,3} = \{t_{10}, t_{11}, t_{12}\} \]
\[ T_{o,4} = \{t_{13}, t_{14}, t_{15}, t_{16}\} \]

**Fairness:** \( v_8 - v_{16} \leq 2 \) and \( v_{16} - v_8 \leq 2 \).

\( (v_i: \text{the number of firings of } t_i.) \)

No acceptable solution without communication!

**Result:**

subsystem 2: broadcast \( t_8 \) and enforce
\[ \mu_5 + \mu_6 + \mu_7 + v_8 - v_{16} \leq 2 \]

subsystem 4: broadcast \( t_{16} \) and enforce
\[ v_{16} - v_8 \leq 2 \]
Conclusion

This paper extends the SBPI to the decentralized setting.

The supervisors can be designed by constraint transformation for:

- no communication
- restricted communication
- minimal communication

This work shows that the decentralized supervision of PNs can be tractable.

On the negative side:

- Our ILP approach is suboptimal.
- Difficult to include permissivity requirements in the ILP.