
At page 189, Lemma 1 is stated as follows:

**Lemma 1:** [Wrong] Let $\mathcal{N} = (P, T, F, W)$ be a Petri net of incidence matrix $D$. Assume that there is an initial marking $\mu_I$ which enables an infinite firing sequence $\sigma$. Let $U \subseteq T$ be the set of transitions which appear infinitely often in $\sigma$.

(a) There is a nonnegative integer vector $x$ such that $Dx \geq 0$, $\forall t_i \in U$: $x(i) \neq 0$ and $\forall t_i \in T \setminus U$: $x(i) = 0$.

(b) There is a firing sequence $\sigma_x$ containing only the transitions with $x(i) \neq 0$, such that $\exists \mu^*_1, \mu^*_2 \in \mathcal{R}(\mathcal{N}, \mu_I)$: $\mu^*_1 \xrightarrow{\sigma_x} \mu^*_2$, each transition $t_i$ appears $x(i)$ times in $\sigma_x$, $\sigma$ can be written as $\sigma = \sigma_a \sigma_x \sigma_b$, and $\mu_I \xrightarrow{\sigma_a} \mu^*_1$.

The correct statement of the lemma is:

**Lemma 1:** [Correct] Let $\mathcal{N} = (P, T, F, W)$ be a Petri net of incidence matrix $D$. Assume that there is an initial marking $\mu_I$ which enables an infinite firing sequence $\sigma$. Let $U \subseteq T$ be the set of transitions which appear infinitely often in $\sigma$. There is a nonnegative integer vector $x$ satisfying (a) and (b) below:

(a) $Dx \geq 0$, $\forall t_i \in U$: $x(i) \neq 0$ and $\forall t_i \in T \setminus U$: $x(i) = 0$.

(b) there is a firing sequence $\sigma_x$ containing only the transitions with $x(i) \neq 0$, such that $\exists \mu^*_1, \mu^*_2 \in \mathcal{R}(\mathcal{N}, \mu_I)$: $\mu^*_1 \xrightarrow{\sigma_x} \mu^*_2$, each transition $t_i$ appears $x(i)$ times in $\sigma_x$, $\sigma$ can be written as $\sigma = \sigma_a \sigma_x \sigma_b$, and $\mu_I \xrightarrow{\sigma_a} \mu^*_1$.

The proof of the lemma in the paper corresponds to this restatement. The mistake in the original statement has no effect on the rest of the paper.

It is interesting to note that part (b) of the original statement is not even true. Indeed, it is not true that “If $x \geq 0$, $Dx \geq 0$, $\forall t_i \in U$: $x(i) \neq 0$ and $\forall t_i \in T \setminus U$: $x(i) = 0$, then there is a firing sequence $\sigma_x$ containing only the transitions with $x(i) \neq 0$, such that $\exists \mu^*_1, \mu^*_2 \in \mathcal{R}(\mathcal{N}, \mu_I)$: $\mu^*_1 \xrightarrow{\sigma_x} \mu^*_2$, each transition $t_i$ appears $x(i)$ times in $\sigma_x$, $\sigma$ can be written as $\sigma = \sigma_a \sigma_x \sigma_b$, and $\mu_I \xrightarrow{\sigma_a} \mu^*_1.”$ This can be seen on a counterexample. The problem arises because the initial marking may cause certain $\sigma_x$ sequences never to be enabled.

In Figure 1(a), note that $Dx = 0$ for $x = [3, 3, 1, 1]$. The marking shown in the figure is the initial marking $\mu_I$. It can be easily seen that we cannot find a sequence $\sigma_x$ that is eventually firable, even
though we can find $\sigma_{x'} = t_1 t_3 t_2 t_4$ for $x' = [1, 1, 1, 1]$ with $Dx' = 0$. Note also that we can find a counterexample that does not involve self-loops, as seen in Figure 1(b). The counterexample would be $x = [3, 3, 1, 1, 3, 3, 1, 1]$ with $Dx = 0$, where the entries of $x$ correspond to $t_1, t_2, t_3, t_4, t'_1, t'_2, t'_3$, and $t'_4$, in this order.