Synthesis of Supervisors Enforcing General Linear Vector Constraints in Petri Nets

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Outline

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• Generality of the constraints
• Supervisor design for fully controllable and observable PNs
• Supervisor design for partially controllable and observable PNs
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Notation

Notation: $\mu$ – the marking, $\mu_0$ – the initial marking, $D$ – the incidence matrix, $q$ – the firing vector, and $v$ – the Parikh vector. Let $\mu_i$ denote $\mu(p_i)$ and $v_j$ denote $v(t_j)$.

The state equation: $\mu = \mu_0 + Dv$. 

\[
\begin{align*}
\mu_0 &= \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}^T \\
v &= \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T \\
q &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T
\end{align*}
\]
Constraint Description

This paper shows that *generalized linear constraints*, involving the marking, the firing vector and the Parikh vector can be enforced as effectively as the linear marking constraints.

The following are defined:

1. Linear Marking Constraints
2. Constraints involving the firing vector and the marking
3. The generalized linear constraints
Constraint Description

1. *Linear Marking Constraints* (also known as *Generalized Mutual Exclusion Constraints*):

   \[ L\mu \leq b \]  
   \[ L\mu_0 \leq b \]

   This requires the initial marking \( \mu_0 \) to satisfy

   and that a transition \( t \) may fire from a marking \( \mu \) iff

   (a) \( \mu \xrightarrow{t} \mu' \)

   (b) \( L\mu' \leq b \)

   In the literature, linear marking constraints have been used to represent

   1. Logical constraints.
   3. Markings for which a PN is deadlock-free/live.
Let $L\mu \leq b$ be $\mu_1 + \mu_3 \geq 1$. Then:

$$L = \begin{bmatrix} -1 & 0 & -1 \end{bmatrix} \quad b = \begin{bmatrix} -1 \end{bmatrix}$$

The incidence matrix is:

$$D = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & -1 & -1 \end{bmatrix}$$

The supervisor has one control place (as $L$ has one row):

$$D_s = -LD = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

The initial marking of the supervisor is

$$\mu_{s0} = b - L\mu_0 = \begin{bmatrix} 1 \end{bmatrix}$$

As $\mu_s = b - L\mu$ for all reachable markings $\mu$, the method is called *supervision based on place invariants*. 

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2. *Constraints involving the marking and the firing vector.*

\[ L\mu + Hq \leq b \]  

(2)

This requires the initial marking \( \mu_0 \) to satisfy

\[ L\mu_0 \leq b \]

and that a transition \( t_i \) may fire from a marking \( \mu \) iff

(a) \( \mu \xrightarrow{t_i} \mu' \)

(b) \( L\mu' \leq b \)

(c) \( L\mu + Hq \leq b \) for \( q(i) = 1 \) and \( q(j) = 0 \) \( \forall j \neq i \).

In the literature, constraints involving the firing vector have been used for

2. The control of railway networks.
3. *The generalized linear constraints:*

\[ L\mu + Hq + Cv \leq b \]  

This requires the initial marking \( \mu_0 \) to satisfy

\[ L\mu_0 \leq b \]

and that a transition \( t_i \) may fire from a current state \((\mu, v)\) iff

(a) \( \mu \xrightarrow{t_i} \mu' \)

(b) \( L\mu + Hq + Cv \leq b \) for \( q(i) = 1 \) and \( q(j) = 0 \) \( \forall j \neq i \).

(c) \( L\mu' + Cv' \leq b \), where \( v' = v + q \).
Constraint Generality

Application of generalized linear constraints:

1. In the literature, the simpler constraints $Cv \leq b$ have been used to specify fairness constraints.
2. We show that any supervisor designed as control places arbitrarily connected to the places of a PN can be described as enforcing constraints $Hq + Cv \leq b$.
3. We show on an AGV coordination example how constraints in the generalized linear form can naturally arise.
The places of any PN can be seen as control places enforcing (3):

\[
\begin{align*}
(p_1) & \quad v_1 \leq 3 \\
(p_2) & \quad v_2 - v_3 \leq 0 \\
(p_3) & \quad -v_2 + v_3 \leq 1
\end{align*}
\]
Given $L\mu + Hq + Cv \leq b$, let:

\[
D_{lc}^+ = \max(0, -LD - C) \tag{4}
\]
\[
D_{lc}^- = \max(0, LD + C) \tag{5}
\]

The supervisor is given by the incidence matrices:

\[
D_c^+ = D_{lc}^+ + \max(0, H - D_{lc}^-) \tag{6}
\]
\[
D_c^- = \max(D_{lc}^-, H) \tag{7}
\]

The initial marking of the supervisor is:

\[
\mu_{c0} = b - L\mu_0 \tag{8}
\]

**Theorem 1.** The supervisor defined by the input and output matrices $D_c^+$ and $D_c^-$ and of initial marking $\mu_{c0}$, enforces $L\mu + Hq + Cv \leq b$ and is least restrictive.
A set of constraints $L\mu + Hq + C\nu \leq b$ is said to be **admissible** if the approach for fully controllable and observable PNs generates a supervisor which never attempts to inhibit plant-enabled uncontrollable transitions and detect closed-loop-enabled unobservable transitions. 

*If $L\mu + Hq + C\nu \leq b$ is not admissible, our approach is to find a set of constraints $L\alpha\mu + H\alpha q + C\alpha\nu \leq b$ such that*

1. $L\alpha\mu + H\alpha q + C\alpha\nu \leq b \Rightarrow L\mu + Hq + C\nu \leq b$
2. $L\alpha\mu + H\alpha q + C\alpha\nu \leq b$ is admissible.

Effective techniques exist for enforcing constraints $L\mu \leq b$ in partially controllable and observable PNs.

Our approach transforms the problem of enforcing $L\mu + Hq + C\nu \leq b$ into the problem of enforcing $L_t\mu_t \leq b$ in a transformed PN.
Illustration of the *C-Transformation*:

This transformation maps

\[ \mu_1 + q_1 + v_2 - v_3 \leq 3 \quad (9) \]

into

\[ \mu_1 + q_1 + \mu_4 - \mu_5 \leq 3 \quad (10) \]

The inverse transformation is possible and maps

\[ \mu_1 - 3\mu_4 + 2\mu_5 + q_1 \leq 5 \quad (11) \]

into

\[ \mu_1 + q_1 - 3v_2 + 2v_3 \leq 5 \quad (12) \]
Illustration of the *H-Transformation*:

This transformation maps

$$\mu_1 + \mu_2 + 2\mu_3 + q_3 \leq 5$$  \hspace{1cm} (13)

into

$$\mu_1 + \mu_2 + 2\mu_3 + 4\mu_5 \leq 5$$  \hspace{1cm} (14)

The term $4\mu_5$ is obtained as follows. Consider firing $t_3$ in the transformed net: $\mu \xrightarrow{t_3} \mu'$. The coefficient $a$ of $t_3$ is to satisfy that

$$a + \mu_1' + \mu_2' + 2\mu_3' = 1 + \mu_1 + \mu_2 + 2\mu_3$$

The inverse transformation can also be defined.
Supervisor Design

Given the PN $\mathcal{N}$ and the set of constraints $L\mu + Hq + Cv \leq b$:

1. Apply the $C$-transformation and then the $H$-transformation. This maps $\mathcal{N}$ to $\mathcal{N}_{HC}$ and $L\mu + Hq + Cv \leq b$ to $L_{HC}\mu \leq b$.

2. Test whether $L_{HC}\mu_{HC} \leq b$ is admissible in $\mathcal{N}_{HC}$. If so, exit, and declare $L\mu + Hq + Cv \leq b$ admissible.

3. Find a set of admissible constraints $L_{HCa}\mu_{HC} \leq b_a$ such that $L_{HCa}\mu_{HC} \leq b_a \Rightarrow L_{HC}\mu_{HC} \leq b$. In case of failure, exit and declare failure to find admissible constraints.

4. Apply the inverse $H$- and $C$-transformations. This maps $L_{HCa}\mu_{HC} \leq b_a$ to $L_a\mu + Haq + Cav \leq b_a$.

**Theorem 2.** $L_a\mu + Haq + Cav \leq b_a$ is admissible, and a supervisor enforcing it enforces also $L\mu + Hq + Cv \leq b$ (that is, $L_a\mu + Haq + Cav \leq b_a \Rightarrow L\mu + Hq + Cv \leq b$.)
The number of AVs in the RA is $v_{13} + v_{14} + v_4 + v_5 - v_9 - v_{10}$ and is limited to $m$.

It is necessary to wait for arbitration when the number of AVs in the RA is $m - 1$ and both a left and a right vehicle attempt to enter the RA.

AVs should not wait for arbitration otherwise.

The arbitration is to be fair (not to favor left or right AVs).
### Example

<table>
<thead>
<tr>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2q_5 + \mu_2 + \mu_7 \leq m - (v_{13} + v_{14} + v_4 + v_5 - v_9 - v_{10}) + 1$ (inadmissible) (15)</td>
</tr>
<tr>
<td>$2q_4 + \mu_3 + \mu_8 \leq m - (v_{13} + v_{14} + v_4 + v_5 - v_9 - v_{10}) + 1$ (inadmissible) (16)</td>
</tr>
<tr>
<td>$mq_3 \leq \mu_3 + \mu_8 + v_{13} + v_{14} + v_4 + v_5 - v_9 - v_{10}$ (impossible) (17)</td>
</tr>
<tr>
<td>$mq_6 \leq \mu_2 + \mu_7 + v_{13} + v_{14} + v_4 + v_5 - v_9 - v_{10}$ (impossible) (18)</td>
</tr>
</tbody>
</table>

$$\mu_2 + \mu_7 \leq 1$$ (admissible) (19)

$$\mu_3 + \mu_8 \leq 1$$ (admissible) (20)

The requirement on the maximum number of AVs in the RA is

$$(v_{13} + v_{14} + v_4 + v_5 - v_9 - v_{10}) \leq m$$ (inadmissible) (21)

### Fairness constraints:

$$(v_3 - v_6) \leq n$$ (admissible) (22)

$$(v_3 - v_6) \leq n$$ (admissible) (23)
Example

The Transformed Constraints

Transformed constraints

\[ 2q_5 + \mu_2 + \mu_5 + \mu_6 + \mu_7 + v_{13} + v_{14} + v_4 + v_5 - v_9 - v_{10} \leq m + 1 \] (24)

\[ 2q_4 + \mu_3 + \mu_5 + \mu_6 + \mu_8 + v_{13} + v_{14} + v_4 + v_5 - v_9 - v_{10} \leq m + 1 \] (25)

\[ v_{13} + v_{14} + v_4 + v_5 - v_9 - v_{10} + \mu_5 + \mu_6 \leq m \] (26)

Relaxed constraints:

\[ mq_3 - \mu_3 - \mu_8 - \mu_5 - \mu_6 - (v_{13} + v_{14} + v_4 + v_5 - v_9 - v_{10}) \leq 0 \] (27)

\[ mq_6 - \mu_2 - \mu_7 - \mu_5 - \mu_6 - (v_{13} + v_{14} + v_4 + v_5 - v_9 - v_{10}) \leq 0 \] (28)
Example

The Supervised PN

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\[ D_c = \]

\[
\begin{bmatrix}
0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & -1 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 & 0 & 0 \\
0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
Final Remarks

**Computability:** This paper has shown that generalized linear constraints can be enforced as effectively as linear marking constraints.

**Generality:** Generalized linear constraints can describe any supervisor consisting of control places connected to the transitions of a plant PN.

**Flexibility:** The technique of this paper transforms the problem of enforcing generalized linear constraints into a problem of enforcing linear marking constraints. Any method can then be used to solve the linear marking constraint problem.

**Implementation:** Software implementation available within the DES software package at: [http://www.nd.edu/~isis/techreports/spnbox/](http://www.nd.edu/~isis/techreports/spnbox/)
References


